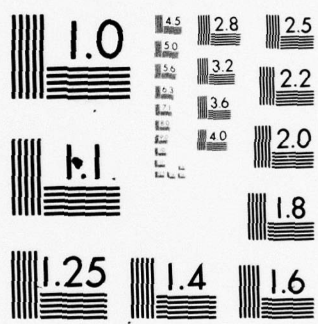
AD-A049 554 WRIGHT STATE UNIV DAYTON OH DEPT OF ADMINISTRATIVE S--ETC F/G 15/5
ALGORITHMS FOR COMPUTING REPAIR SCHEDULES IN PROBABILISTIC ENVI--ETC(U)
APR 77 W S DEMMY AFOSR-76-3011
UNCLASSIFIED WP-76-3011-03 AFOSR-TR-77-1294 NL

1 OF 1
ADA049 554

END
DATE
FILMED
3 -78
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A049554

AD NO.

DDC FILE COPY

4

AFOSR-TR- 77-1294

Approved for public release;
distribution unlimited.

COLLEGE of BUSINESS and ADMINISTRATION

Wright State University
dayton, ohio

4

6

ALGORITHMS FOR

COMPUTING REPAIR SCHEDULES

IN

PROBABILISTIC ENVIRONMENTS,

9 Interim rept.,

by

10 W. Steven Demmy

12 32p.

11 April 1977

14 WP-76-3Ø11-Ø3

15 AFOSR-76-3Ø11

Working Paper 76-3011-03
Department of Administrative Science and Finance
Wright State University

16 23Ø4

17 A5

18 AFOSR

19 TR-77-1294

410 555

mt

ABSTRACT

In this paper, we consider an organization that repairs or overhauls a single type of complex equipment such as aircraft transmitters, navigational computers, or electrical generating units. The organization inventories the repaired assets to meet probabilistic demands for serviceable items. Additional serviceable stocks may also be purchased from outside vendors if the repair process does not satisfy all demands.

The paper considers the problem of computing jointly optimal procurement and repair schedules for such a system. Cost elements explicitly considered include set-up costs, learning curve effects, expediting and inventory holding costs, and penalties for incurring shortages or for modifying the schedule from previously established levels. Multi-period lead times, obsolescence and time-discounting effects are also considered.

The paper presents two solution procedures for this scheduling problem . First, we present a general dynamic programming formulation that involves two state varables and two decision variables for each planning period. Next, we present the Support Level Algorithm (SLA) which determines an optimal "Manne-type" schedule. In two important special cases, the SLA simplifies to a form quite similar to the well-known Wagner-Whitin algorithm.

Table of Contents

		Page
I.	Introduction	1
II.	A Single Item Model	5
III.	The Support Level Algorithm	17
	Bibliography	

I. Introduction

Consider an organization such as that illustrated in Figure 1 that repairs or overhauls complex assemblies and places the repaired assets in inventory to meet demands for serviceable items. In this system, initial serviceable stocks are procured from an outside vendor, and additional serviceable stocks may be procured from the vendor if it is not possible or is uneconomical to satisfy all customer demand through operation of the repair facility.

As illustrated in the figure, many but not necessarily all demands are accompanied by a "turn-in" of a reparable asset. Generally, the total number of turn-ins during a given period of time will differ from the total demand for serviceable assets during the period due to:

a. condemnation or loss of the asset at the customer level due to wearout, accident, theft, or other causes,

b. increased levels of customer operation, resulting in an initial demand for serviceable assets to increase both the number of serviceable units operating in the field and the customer's inventory of spares of these items,

c. decreased levels of customer operation, resulting in turn-ins without corresponding demands for serviceable assets.

Once a reparable asset becomes available, it is transported to a manufacturing facility for repair. These assets are first stored in an unserviceable item warehouse. When sufficient reparable assets are

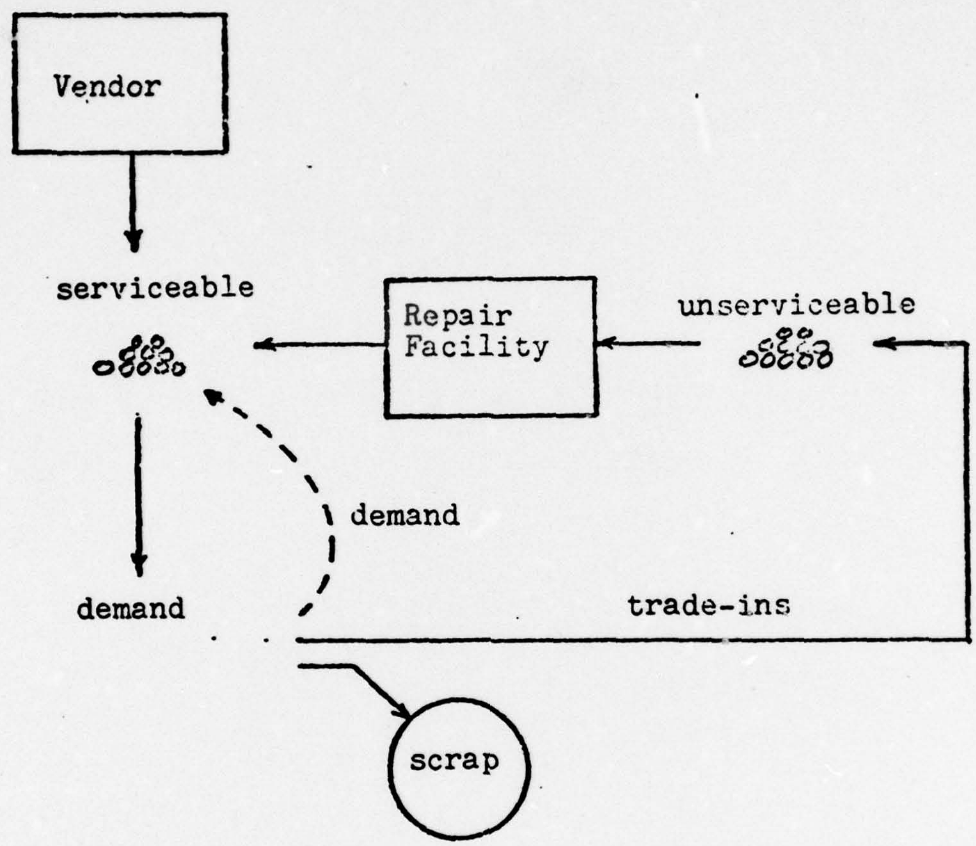


Figure 1. A Simple Inventory and Repair System

available to justify setup of a repair line and when sufficient facilities, manpower, and material are available to support the repair effort, a batch of items are inducted into the repair process. After a suitable flow time, the items are repaired and moved to the serviceable item inventory. In this paper, we assume that the repair process converts reparable assets to a "like-new" condition, i.e., repaired assets are functionally indistinguishable from new items.

Let us assume that the manager of this hypothetical organization must devise detailed procurement and repair schedules for each item in the system. These schedules establish, respectively, the specific quantities of each item to be procured and repaired for each of several future periods. The development of these schedules is complicated by the limited availability of reparable assets and by physical and administrative limitations on the manpower, equipment, facilities, and funds available to support the repair and procurement efforts. We assume that these schedules are to serve as the basis of long-term procurement contracts with the external vendor and with suppliers of the repair facility. Hence, once these schedules are established, it is extremely difficult if not impossible to modify them, at least in the near term.

Our objective is to assist the manager of this hypothetical system by identifying a set of schedules that minimize the discounted expected costs of operating this complex system, consistent with the operating constraints described above. In this paper, we develop algorithms for solving single-item scheduling problems, i.e. problems in which there is no significant competition among inventory items for production resources. In a later paper, we will show how large scale optimization techniques may be employed to reduce the constrained multi-item scheduling problem to a series of the single-item problems that are discussed here.

The paper consists of three sections. In Section II, we develop a general model for the repair scheduling problem, and we then present a general dynamic programming prodedure for solving this problem.

Section III presents the Support Level Algorithm (SLA), which determines an optimal "Manne-type" schedule. In two important special cases, the SLA simplifies to a form quite similar to the well-known Wagner-Whitin algorithm. We conjecture that this algorithm should produce near-optimal solutions to the original scheduling problem, but at a lower computational cost. However, this conjecture is yet to be tested computationally.

II. A Single Item Model

In this section we develop a technique for computing jointly optimal procurement and repair schedules for a relatively simple single item inventory/repair system. Later in this section, we discuss how the basic technique may be extended to deal with more complex single-item systems.

We assume that:

II.1. All unsatisfied demand is backordered until filled; no sales are lost.

II.2. Sufficient manpower, equipment, and funding is available to support any "reasonable" procurement or repair schedule.

II.3. Both procurement and repair leadtimes are one period.

Let $T_1, T_2, \ldots, T_I$ denote the I times within the planning horizon that inventory reviews take place. For convenience, we assume $T_1 = 0$ and we refer to the interval between the times T_i and T_{i+1} as period i. We define:

w_i = quantity of serviceable assets available at the beginning of period i. Negative values of w_i denote backorders outstanding at the beginning of the period,

q_i = quantity of serviceable assets procured from the external vendor for delivery during period i;

r_i = quantity of reparable assets scheduled to complete repair during period i,

d_i = demand for serviceable assets during period i,

where d_i denotes a random variable with distribution function $F_i(d_i)$, and d_i, d_j are statistically independent for all $i \neq j$.

The above variables are related by the following material balance equation,

$$(1) \quad w_{i+1} = w_i + q_i + r_i - d_i$$

where w_1, the initial on-hand serviceable inventory, is assumed known.

<u>Cost of Procurement, Repair, Holding, and Shortage</u>. Let us now consider the cost components of operating this system. We define:

$\overline{CP}_i(q_i)$ = cost of procuring q_i units for delivery at the end of period i,

$\overline{CR}_i(r_i)$ = cost of completing repair of r_i assets during period i,

$CH_i(w_{i+1})$ = cost of holding w_{i+1} units of stock at the end of period i (i.e. the beginning of period i + 1). If w_{i+1} is negative, $CH_i(w_{i+1})$ denotes a penalty charge for backorders,

where $\overline{CP}_i(q_i)$, $\overline{CR}_i(r_i)$, and $CH_i(w_{i+1})$ are known, real, single-valued functions representing the present value of all relevant costs discounted to the beginning of period i. For example, $\overline{CP}_i(q_i)$ might represent a fixed cost to order plus a series of price breaks. The function $\overline{CR}_i(r_i)$ might quantify a set-up time and a learning curve effect, while the function $CH_i(w_{i+1})$ might represent a linear charge per unit for holding stock, and another, higher linear charge for each unit backordered. These examples are illustrated in Figure 2a, 2b, and 2c.

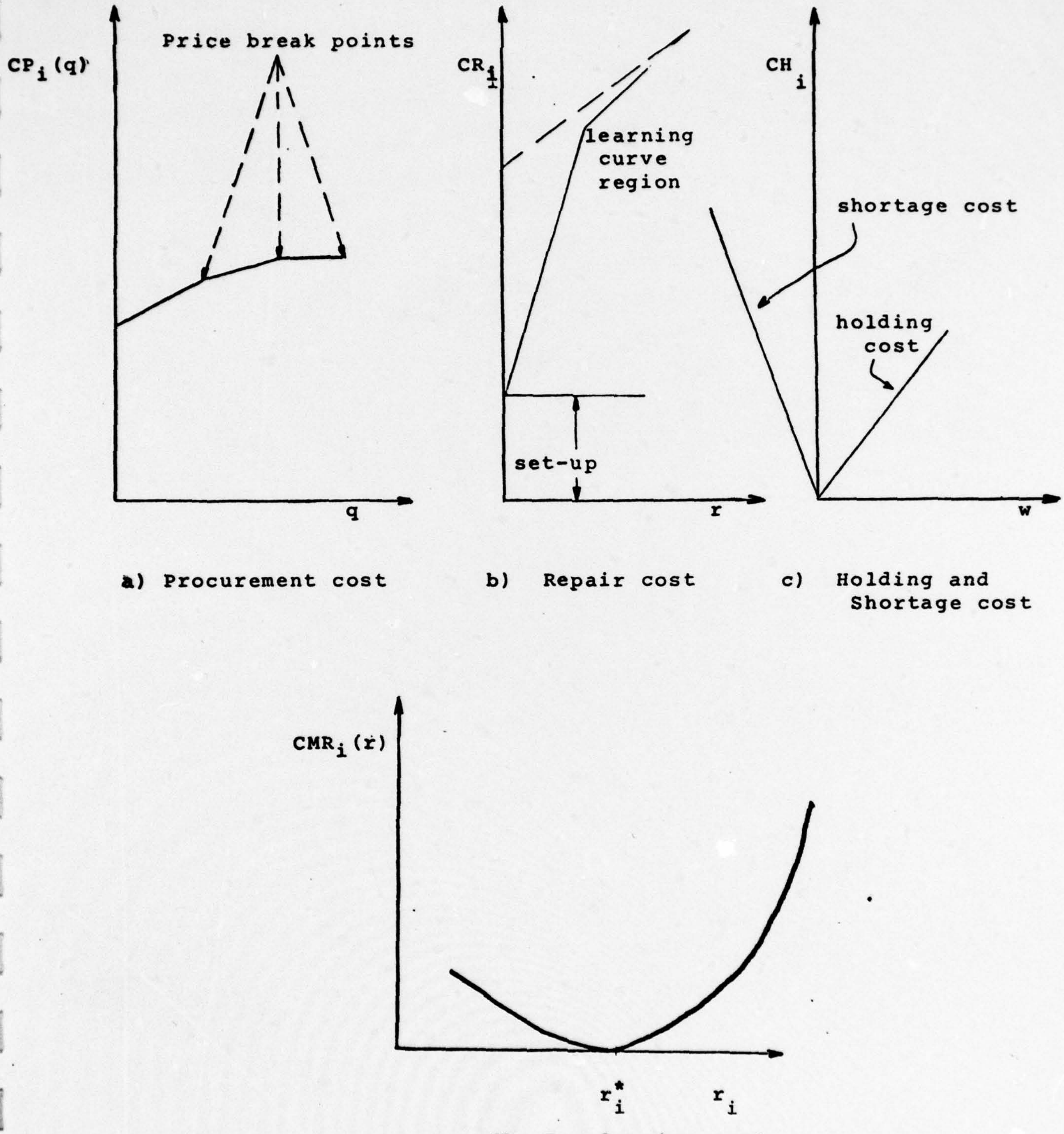


Figure 2. Representative procurement, repair, holding and shortage costs, and replanning costs.

Cost of Replanning

As noted earlier, we assume that the detailed schedules will serve as a basis for procurement contracts with external suppliers and for training and material planning for the repair organization itself. Hence, once these plans are established, it is extremely difficult to change them, at least in the short term. Figure 2d illustrates one cost structure that might be related to changes in an established plan. In the figure, r_i^* denotes the number of units that were originally planned to be repaired in period i, and $\overline{CMR}_i(r_i)$ denotes the expected cost of modifying the repair quantity from the original value of r_i^* units to the new value of r_i units. Hence, $\overline{CMR}_i(r_i)$ represents costs required to replan the repair effort, and to take those steps required to implement the revised plan. For example, Figure 2d represents a situation in which replanning costs increase rapidly as the planned repair quantity is increased. This effect may occur when many expediting actions must be taken to insure adequate material and manpower resources to satisfy the increased repair program. Figure 2d also indicates that replanning costs increase if the scheduled repair is reduced below the originally planned level. This is due to efforts required to reassign manpower to other projects, and to inventory holding and material handling costs incurred when staged materials are held over for use in later periods.

Note that we may form a composite function

$$CR_i(r_i) = \overline{CR}_i(r_i) + \overline{CMR}_i(r_i)$$

representing the total of repair and replanning costs to be incurred if r_i units are scheduled. Similarly, we may form a composite function

$$CP_i(q_i) = \overline{CP}_i(q_i) + \overline{CMQ}_i(q_i)$$

representing the sum of all procurement costs--including replanning costs $CMQ_i(q_i)$--associated with a new scheduled procurement of q_i units.

__Costs of Expediting Reparable Asset Deliveries__. Now let us consider costs of insuring sufficient reparable assets are available to support an established repair schedule. As noted above, turn-ins of reparable assets are returned to the manufacturing facility and stored until needed. We assume that, if necessary, delivery of these assets to the repair facility may be expedited to insure sufficient reparable assets are available to support the repair schedule. In general, such expediting costs will increase significantly with the amount of "speed-up" in deliveries. To quantify these costs, let y_i denote the cumulative number of reparable assets scheduled to be started into repair in periods 1 through i - 1, where $y_1 = 0$ by definition and

(2) $$y_{i+1} = y_i + r_i \qquad \text{for} \quad 2 \leq i \leq I$$

Finally, let

$CY_i(y_i, r_i)$ = cost of insuring __at least__ r_i reparable assets are available at the beginning of period i given y_i assets have started repair in previous periods.

For example, suppose there are u reparable assets on hand at time T_1, and let $t_1, t_2, \ldots, t_n$ denote, respectively, the arrival times of the first, second, ..., and $n^{\underline{th}}$ reparable assets if no expediting actions are taken. If we assume the cost to expedite the $n^{\underline{th}}$ reparable asset is proportional to the speed-up in the arrival time of that asset, the cost C_{in} of assuring the $n^{\underline{th}}$ reparable asset arrives by the beginning of period i is given by

$$C_{in} = \begin{cases} CT.(t_n - T_i) & \text{if } t_n > T_i \\ 0 & \text{otherwise} \end{cases}$$

where CT denotes the cost per unit time of the speedup. Hence, the expected expediting cost in period i is given by

$$CY_i(y,r) = \sum_{n=y-u+1}^{y-u+r} CT.(t_n - T_i) f'_n(t_n) dt_n$$

where $f'_n(t_n)$ denotes the probability density function associated with t_n.

Another useful form of the $CY_i(y,r)$ function is derived as follows: Suppose the expediting cost in a given period is proportional to the number of assets expeditied. Let $p_i(n_i)$ denote the probability that exactly n_i reparable assets will arrive in periods 1 thru i - 1 unless expediting actions are taken to assure more arrivals. If y_i+r_i assets are required to support repair in periods 1 through i, no expediting will be required in period i if $n_i = y_i+r_i-u$ (i.e., if reparable arrivals exceed net requirements). On the other hand, if $y_i - u < n < y_i + r_i - u$ then $(y_i+r_i-u)-n_i$ assets must be expedited to support the repair schedule. Finally, if $n_i < y_i-u_i$, then (y $(y_i+r_i-u\ )-(y_i-u) = r_i$ additional assets must be expedited in period i. This latter case results from the assumption that the delivery of y_i units is assured, if necessary, by actions taken prior to period i. Hence, under these assumptions, the expected cost of assuring an additional r_i assets are available at the beginning of period i is given by

$$CY_i(y,r) = CE*[\sum_{n=y-u+1}^{y-u+r} (y+r-u-n_i)p_i(n_i) + \sum_{n=0}^{y-u} r\ p_i(n_i)]$$

where CE denotes the cost per unit expedited.

In those cases in which $p_i(n_i)$ may be approximated by a Normal distribution with known mean and variance, formulas given by Brown [4,p.93] may be used as a convenient way to compute $CY_i(y,r)$.

Operating Costs in Period i. From the above discussion the total cost TC_i of operating the inventory/repair system during period i, given the item is not obsolete at the end of period i, is given by

(3) $$TC_i = CP_i(q_i) + CR_i(r_i) + CH_i(w_{i+1}) + CY_i(y_i,r_i)$$

Cost of Obsolence. Let B_i denote a survival probability associated with period i, i.e., let B_i denote the conditional probability that the item is not obsolete at the beginning of period i+1 given it is not obsolete at the beginning of period i. For the moment, we assume that if an item becomes obsolete during period i, the procurement and repair actions then in effect may be canceled at a net cost of $COP_i(q_i)$ and $COR_i(r_i)$, respectively, and that available inventories of the obsolete item are immediately disposed of at a net cost of $COW_i(w_i)$. If more than one period is required to complete these obsolescence-imposed actions (or if period i is sufficiently long for discounting to be significant) we assume $COP_i(q_i)$, $COR_i(r_i)$ and $COW_i(w_i)$ denote the present value of all relevant costs at the beginning of period i. Hence, the total cost $TCOB_i$ of operating the inventory/repair system during period i given obsolescence occurs within the period is given by

(4) $$TCOB_i = COP_i(q_i) + COR_i(r_i) + COW_i(w_i)$$

Total Expected Cost in Period i. From the above discussion, the expected cost of operating the inventory system during period i, ETC_i, given the item is not obsolete at the beginning of the period, is given by

(5) $$ETC_i = B_i E\ TC_i + (1 - B_i) E\ TCOB_i$$

where E . denotes the expectation operator. Substituting (3) and (4) into (5), we obtain

(6) $$ETC_i = B_i[CP_i(q_i) + CR_i(r_i) + CH_i(w_{i+1}) + CY_i(y_i,r_i)] + (1-B_i)[COP_i(q_i) + COR_i(r_i) + COW_i(w_i)]$$

Observe that since $CP_i(q_i)$, $CR_i(r_i)$, $CY_i(y_i, r_i)$, $COP_i(q_i)$ and $COR_i(r_i)$ contain no random elements, equation (6) may be written as

(7) $$ETC_i = CP_i'(q_i) + CR_i'(r_i) + CY_i'(y_i, r_i) + B_i E(CH_i(w_{i+1})) + (1-B_i)E(COW_i(w_i))$$

where

(8) $$CP_i'(q_i) = B_i CP_i(q_i) + (1-B_i)COP_i(q_i)$$

(9) $$CR_i'(q_i) = B_i CR_i(r_i) + (1-B_i)COR_i(r_i)$$

(10) $$CY_i'(y_i,r_i) = B_i CY_i(y_i,r_i)$$

Observe that equation (1) implies

(11) $$w_{i+1} = w_i + \sum_{t=1}^{i} (q_t+r_t) - \sum_{t=1}^{i} d_t$$

for $i > 1$. Now let $z_1 = w_1$ and define

(12) $$z_i = w_i + \sum_{t=1}^{i-1} (q_t+r_t)$$

for $i > 1$ and

(13) $$D_i = \sum_{t=1}^{i} d_t$$

for $i \geq 1$. Substituting (12) and (13) into (11), we obtain

(14) $$w_{i+1} = z_i + q_i + r_i - D_i$$

which may also be written as

(15) $$w_{i+1} = Z_{i+1} - D_i$$

Note that z_{i+1} represents the total number of serviceable assets available to meet demands in periods 1 through i, while D_i denotes the cumulative demand in these periods.

Since D_i is the sum of i random variables $d_1, d_2, \ldots, d_i$, the distribution function of D_i, $\Phi_i(D_i)$, is the i-fold convolution of the distribution functions $F_t(d_t)$, t=1,2,...,i. Hence, knowing $F_t(d_t)$, we may evaluate the last two terms in (7).

Specifically,

(16) $$E\,\{CH_i(\omega_{i+1})\} = ECH_i(z_i + q_i + n_i)$$

where

(17) $$ECH_i(\alpha) = \int_0^{\alpha} CH_i(\alpha - D_i)\,\Phi_i^{\prime}(D_i)$$

where $\Phi_i^{\prime}(D_i)$ denotes the probability density function associated with $\Phi_i(D_i)$.

Also,

(18) $$E\,\{COW_i(\omega_i)\} = ECOW_i(z_i)$$

where

(19) $$ECOW_i(z) = \int_0^{z} COW_i(z-D)\,\Phi_{i-1}^{\prime}(D)$$

Hence, we may write equation (7) as

(20) $$ETC_i = CP_i^{\prime}(q_i) + CR_i^{\prime}(r_i) + CY_i^{\prime}(y_i, r_i) + CH_i^{\prime}(z_i, q_i, r_i)$$

where

(21) $$CH_i^{\prime}(z_i, q_i, r_i) = B_i ECH_i(z_i + q_i + r_i) + (1-B_i) ECOW_i(z_i)$$

and where y_i and z_i are defined by equations (2) and (12).

<u>The Single-Item Problem.</u> Let $\eta_i, \eta_i \geq 0$, denote the discounted value at the beginning of period i of one cost unit at the beginning of period i+1. For notational convenience, let

(22) $\alpha_1 = 1$

and

(23) $\alpha_i = \prod_{t=1}^{i-1} \eta_t B_t$

for $i \geq 1$. We wish to determine fixed procurement and repair schedules for the item under consideration so as to minimize the expected discounted costs of operation the inventory system over a given planning horizon of I periods. In symbols, our problem is to determine $Q = (q_1', q_2, \ldots, q_I)$ and $R = (r_1', r_2, \ldots, r_I)$ so as to minimize $E\{TC\}$,

(24) P1: $E\{TC\} = E\ \{\sum_{i=1}^{I} \alpha_i\ TC.\}$

(25) $$= \sum_{i=1}^{I} \alpha_i [CP_i'(q_i) + CR_i'(r_i) + CY_i'(y_i, r_i) + CH_i'(z_i, q_i, r_i)]$$

subject to (2) and (12). We will call this problem P1.

This problem may be solved by a straight-forward I-stage dynamic programming computation. To see this, let

$Z_i(y_i, z_i)$ = minimum expected cost in periods i, i=1,...,I assuming the planned serviceable inventory at the beginning of period i is z_i, that y_i unserviceable assets must be provided to support the repair schedule in periods 1 through i-1, and optimal procurement and repair quantities are scheduled in period i and all subsequent periods.

Hence, the set of values Q,R which minimize $Z_1(y_1,z_1) = Z_1(0, w_1)$ solves P1. Following the arguments in Nemhauser [18, p 155], the required values may be determined by the backward recursive computation of $z_i(y_i,z_i)$ for all feasible (y_i,z_i) - pairs for i = I, I-1,...,2,1, where

(26) $f_{I+1}(y_{I+1},z_{I+1}) = 0$ for all y_{I+1}, z_{I+1}

and

(27) $$Z_i(y_i,z_i) = \underset{q_i r_i}{\text{Min}} \{ CP_i'(q_i)+CR_i(r_i)+CY_i(y_i,r_i) + CH_i'(z_i,q_i,r_i)$$

$$+n_i B_i Z_{i+1}(y_i+r_i,z_i+q_i+r_i)\} \quad \text{for } 1\ i\ I.$$

Once $Z_i(.,.)$ has been computed, the required procurement and repair schedules may be determined by the usual dynamic programming backtracking calculations.

Multi-Period Leadtimes. *The model presented above may be extended to* represent multiple-period procurement or repair leadtimes. Let LR and LQ denote, respectively, the number of periods between the start of procurement and repair actions and the completion of those actions. Let $CR_i^t\ (r_{i+t-1})$ denote the cost incurred during period i to support the $t^{\underline{th}}$ period of repair for the r_{i+t-1} units scheduled to complete repair in period i+t-1, and let $CP_i^t(q_{i+t-1})$ denote a similarly defined procurement cost function. Let $COP_i^t(q_{i+t-1})$ and $COR_i^t(r_{i+t-1})$ denote, respectively, the net cost of terminating procurement or repair actions that are in the $t^{\underline{th}}$ period of completion during period i should the item become obsolete during the period, and let $COW_i(w_i)$ and $CY_i(y_i r_i)$ be defined as above. Note that when multiple-period repair flow times are involved, r_i reparable carcasses must be available in period i -LR if r_i units are to complete repair in period i. This fact must be considered in computing numeric values for $CY_i(.)$.

The revised problem, P2, is to determine fixed joint procurement and repair schedules so as to minimize ETC, the expected total cost of operating the system over I periods, where

$$(28)\quad ETC = \sum_{i=1}^{I} \alpha_i [B_i \langle \sum_{t=1}^{LQ} CP_i^t (q_{i+t-1}) + \sum_{t=1}^{LR} CR_i^t (r_{i+t-1}) +$$

$$CY_i(y_i, r_i) + ECH_i (z_i + q_i + r_i)) + (1-B_i) \langle \sum_{t=1}^{LQ} COP_i^t (q_{i+t-1})$$

$$+ \sum_{t=1}^{LR} COR_i^t (q_{i+t-1}) + ECOW_i (w_i))]$$

subject to (2) and (12). We assume that $q_1, q_2, \ldots, q_{LQ}$ and $r_1, r_2, \ldots, r_{LR}$, the procurement and repair quantities already in process at the beginning of period 1, are known.

Expanding (28) and regrouping terms, we obtain

$$(29)\quad ETC = \sum_{i=1}^{I} [CP_i''(q_i) + CR_i''(r_j) + CY_i''(y_i, r_i) + CH_i''(z_i, q_i, r_i)]$$

where

$$(30)\quad CP_i''(q_i) = \sum_{t=1}^{LQ} \alpha_{i-LQ+t} [B_{i-LQ+t} CP^t_{i-LQ+t}(q_i) + (1-B_{i-LQ+t})\, COP^t_{i-LQ+t}(q_i)]$$

$$(31)\quad CR_i''(r_i) = \sum_{t=1}^{LR} \alpha_{i-LR+t} [B_{i-LR+t}\, CR^t_{i-LR+t}(r_i) + (1-B_{i-LR+t}\, COR^t_{i-LR-t}(q_i)]$$

$$(32)\quad CY_i''(y_i, r_i) = \alpha_i B_i CY_i(y_i r_i)$$

$$(33)\quad CH_i''(z_i, q_i, r_i) = \alpha_i [B_i ECH_i(z_i + q_i + r_i) + (1-B_i) ECOW_i(z_i)]$$

Observe that (29) has the same form as (25). Hence, the dynamic programming calculation described above may also be used to solve the multi-period leadtime problem, P2.

III. The Support-Level Algorithm

Although the dynamic programming calculation (27) provides an optimum solution to the problem (P1), substantial computational costs may be required if the number of (q_i,r_i)-pairs which must be evaluated is large One method of reducing these costs is to reduce the set of (q_i,r_i)-pairs explicitly considered in the recursive computation. In many situations, the savings in computational costs obtained through this device may more than compensate for the suboptimality of the resulting solution.

One particularly promising simplification is based upon an extension of ideas employed in the classical dynamic lot-size model to the fixed-schedule, probabilistic-demand situation considered here. Recall that Wagner and Whitin [30] devised an algorithm to determine the optimum solution $Q^*=(q^*_1,q^*_2,\ldots,q^*_I)$ to the problem of minimizing C(Q),

$$(34)\quad C(Q) = \sum_{i=1}^{I} [CP_i(q_i) + CH_i(w_{i+1})]$$

subject to

$$(35)\quad w_{i+1} = w_i + q_i - d_i$$

$$(36)\quad w_i \geq 0$$

where w_1 is assumed known, demand d_i is deterministic, and the cost functions $CP_i(q_i)$ and $CH_i(w_{i+1})$ are concave. Observe that (34)-(36) is a special case of problem pl. Manne (1958) observed that an optimum solution to the problem (34)-(36) is characterised by the fact that procurement deliveries (i.e. $q_i>0$) may take place in a period only if no inventory enters that period. Thus, if a delivery occurs in period i and next in period s-i, and if these delivery

decisions are components of the optimum solution to the dynamic lot size problem (34-(36), then the delivery quantity q^*_i is given by

$$(37) \quad q^*_i = \sum_{t=1}^{s-1} d_t$$

where d_t denotes the requirement in period t. That is, under an optimal policy for the dynamic lot size problem, the quantity delivered in period i is just sufficient to satisfy the known requirements in periods i, i+1,...,s-1. Wagner and Whitin provide an efficient algorithm that exploits these properties.

The Support Level Algorithm (SLA) defined below is an application of ideas employed in the Wagner-Whitin algorithm to the problem pl. The algorithm consists of several phases. During phase h, we compute the minimum cumulative number of serviceable assets x_i which must be delivered to serviceable stocks in periods 1 through i to guarantee there is no more than an α_h- probability of a stockout at the end of period i. Specifically, given the initial value of on hand stocks w_1, we compute

$$(38) \quad x_i = \min_{x \geq o} (x \mid \Phi_i(x+w_1) \geq \alpha_h)$$

where $\Phi_i(x)$ denotes the probability distribution function of D_i (i.e. $\Phi_i(x)$ = Prob $(D_i \leq x)$) and α_h is a specified constant. We call α_h the "support level" associated with the cumulative delivery schedule x = $(x_1, x_2, \ldots, x_I)$. Now let $D^* = (d^*_1, d^*_2, \ldots, d^*_I)$ denote the schedule of incremental values of X; i.e. let

$$(39) \qquad d^*_1 = x_1$$
$$d^*_i = x_i - x_{i-1}$$

for $i > 1$.

The SLA algorithm restricts its attention to delivery schedules constructed from D^* with the properties of an optimal solution to the classical dynamic lot size model, i.e., the algorithm restricts its attention to (q_i, r_i)-pairs such that in period i

$$(40) \qquad q_i + r_i = \sum_{t=i}^{s} d^*_t$$

for some $s > i$, and r_i is restricted to the set $(d^*_i, d^*_i + d^*_{i+1}, \ldots d^*_i + d^*_{i+1}, \ldots + d^*_I)$. Further, we assume that if D_{is} units are scheduled to be delivered during period i (from repair and/or procurement,) where

$$(41) \qquad D_{is} = \sum_{t=r}^{s-1} d^*_t,$$

then no other deliveries will be scheduled until period s. Now, let $g_{ist}(y_i)$ denote the sum of a) costs of scheduling D_{is} units to complete repair in period i, b) costs of scheduling D_{st} units to be delivered from the outside vendor in period i, c) holding and shortage costs in periods i, i+1,...,t-1 given y_i assets have started into repair in periods 1 through i-1. Hence, by definition

$$(42) \qquad g_{ist}(y_i) = CP''_i(D_{st}) + CR''_i(D_{is}) + CY''_i(y_i, D_{is}) + \sum_{k=1}^{t-1} CH''_k(z_i + D_{1t})$$

where z_i is defined as above. Observe, however, that the restriction (39) and (40) and equation (12) imply that if a delivery is scheduled in period i,

$$(43) \qquad z_i = w_1 + \sum_{t=1}^{i-1} d^*_t = w_1 + D_{1t}$$

Hence, since $D_{1i} + D_{it} = D_{1t}$, equation (42) may be written as

$$(44) \quad g_{ist}(y_i) = CP''_i(D_{st}) + CR''_i(D_{is}) + CY''_i(y_i, D_{is}) + \sum_{k=1}^{t-1} CH''_k(w_1 + D_{1t})$$

Now let $f_i(y_i)$ denote the minimum cost over periods i, i+1,...,I given a delivery occurs in period i and y_i units have started into repair in periods 1 thru i-1. Defining $f_{I+1}(y_{I+1}) = 0$ for all y_{I+1}, the backward recursive computation of

$$(45) \qquad f_i(y_i) = \min_{i \leq t \leq I+1} \left\{ \min_{i < s < t} \left[g_{ist}(y_i) + f_t(y_i + D_{is}) \right] \right\}$$

for all possible values of y_i yields an optimal schedule subject to condition (40).

With the above introduction, we may state the Support Level Algorithm as follows:

A. Given: H, the number of support levels to be considered;

α_h, for h = 1,2,...,H;

$\Phi_i(\cdot)$, for i = 1,2,...,I; and

w , the initial on hand serviceable inventory.

1. Set h=1 and FMIN = ∞

2. Compute X and D* using equations (38) and (39).

3. Set $i=I$.

4. Evaluate $f_i(y_i)$ using (45) for $y_i=0,1,2,\ldots,y_i'$, where y_i' is a reasonable upper bound on y_i.

   Let $S_i(y_i)$ and $T_i(y_i)$, respectively, denote the values of s and t which produce the minimum $f_i(y_i)$ in (45).

5. If $i=1$, go to step 6; otherwise replace i by $i-1$ and repeat step 4.

6. The current value of $f_1(0)$ is the minimum discounted cost associated with support level h. If $f_1(0)<\text{FMIN}$, go to step 7; otherwise, go to step 8.

7. We wish to record the schedule associated with $f_1(0)$ as the best schedule found to this point in the computation. To do this, perform the following steps:

   a. Set $i=1$ and let $y^*=0$

   b. Set $s=S_i(y^*)$ and $t=T_i(y^*)$

   c. Then $r=D_{is}$, $q=D_{st}$, and $R_k^*=q_k^*=0$ for $k=i+1, i+2,\ldots,t-1$.

   d. If $t=I+1$, go to step 8; the best schedule has been identified. Otherwise, set $i=t$ and $y^*=y^*+D_{is}$ and go to step 7b.

8. If $h=H$ stop; the set (r_i^*,q_i^*) with associated cost FMIN is the minimum cost schedule satisfying SLA restrictions. Otherwise, set $h=h+1$ and go to step 2.

The Repair Algorithm. Observe that if procurement is not possible, equations (2) and condition (40) imply that if a repair completion is to occur in period i, then $Y_i=D_{1i}$.

Hence, in this case, in step 4 of the SLA we need consider only one value for the state variable, i.e. $y_i = D_{1i}$. This significantly reduces the computational burden associated with SLA calculations. In essence, this simplification reduces the SLA calculations to be quite similar to the classical Wagner-Whitin algorithm.

The Procurement Algorithm. Now suppose that procurement is possible, but repair is not a feasible alternative. In this case, equation (2) and condition (40) imply that y_i must equal 0. Hence, in this case as above, we need consider only one value for y_i in step 4 of the SLA; namely, $y_i=0$.

BIBLIOGRAPHY

1. Allen, S.G., and D.A. D'Esopo, "An Ordering Policy for Repairable Stock Items," Operations Research, v16, n3, May-June 1968, pp. 669-674.

2. Brooks, R. and A. Geoffrion, "Finding Everett's Lagrange Multipliers by Linear Programming," Operations Research, v14, 1966, pp. 1149-1153.

3. Brown, G.F., T.M. Corcoran, and R.M. Lloyd, A Dynamic Inventory Model with Delivery Lag & Repair, Center for Naval Analyses, University of Rochester, Aug 1, 1969.

4. Brown, R.G., Decision Rules for Inventory Management, Holt, Rhinehart and Winston, New York, 1967, p. 93.

5. Demmy, W.S., Allocation of Spares and Repair Resources to a Multi Component System, AFLC Report 70-17, Office of DCS/Comptroller, Headquarters Air Force Logistics Command, Wright-Patterson Air Force Base, Ohio.

6. Dzielinski, B.P., C.T. Baker, and A.S. Manne, Simulation Tests of Lot Size Programming, Management Science, v9, n2, January 1963, pp. 229-258.

7. Dzielinski, B.P. and R.E. Gomory, Optimum Programming of Lot Sizes, Inventories, and Labor Allocations, IBM-ASDD TR 17-120, May, 1964.

8. Dzielinski, B.P. and R.E. Gomory, "Optimum Programming of Lot Sizes, Inventories, and Labor Allocations," Management Science, v11, n9, July 1965, pp. 874-890.

9. Evans, J.P. and F.J. Gould, "Application of the GLM Technique to a Production Planning Problem," Naval Research Logistics Quarterly, v18, n1, March 1971, pp. 59-74.

10. Everett, H., "Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources," Operations Research, v11, 1963, pp. 399-417.

11. Freiheit, James E., A Continuous Review Model for the Reparable Item Inventory System, master's thesis, Naval Postgraduate School, Monterey, California, June 1967, 47p. (AD 817 560)

12. Hadley, G. and T.M. Whitin, "Budget Constraints in Logistics Models," Naval Research Logistics Quarterly, v8, n3, Sept 1961, pp. 215-220.

13. Hastings, David A., A Constrained Periodic Review Model for a Probabilistic Reparable-Item Inventory System, Master's Thesis, U.S. Naval Postgraduate School, Monterey, California, June 1967, 31p. (AD 823 751)

14. Hoekstra, D., Multi-Level Inventory Systems: The Case of the Spare Aircraft Engine, Frankford Arsenal, Philadelphia, Pa., 1964, 26p. (AD 615 892).

15. Kortanek, K.O., D. Sodaro and A.L. Soyster, "Multi-Product Production Scheduling Via Extreme Point Properties of Linear Programming," Naval Research Logistics Quarterly, v15, n2, June, 1968, pp. 287-300.

16. Kortanek, K.O. and A.L. Soyster, "On the Status of Some Multi-Product Multi-Period Production Scheduling Models," Management Science, v17, n8, April 1971, pp. 560-561.

17. Krause, W.K., Optimal Multi-Echelon Stockage Policies with Constraints, Inventory Research Office, United States Army Logistics Management Center, Fort Lee, Va., Feb 1970, 52 pp.

18. Nemhauser, G.L., Introduction to Dynamic Programming, John Wiley and Sons, 1967.

19. Rao, M.R., Multi-Commodity Warehousing Models with Cash-Liquidity Constraints - A Decomposition Approach, Management Sciences Research Report No. 141, Graduate School of Industrial Administration, Carneigie-Mallon University, Pittsburg, September, 1968, 175 pp. (AD 680 617.)

20. Rose, M., An Investment Model for Reparable Assets: The F-4 Case, Report No. 31, Institute of Naval Studies, Center for Naval Analysis, Arlington, Va., July 1969, 159., (AD 694 946).

21. Schrady, D.A., "A Deterministic Inventory Model for Reparable Items," Naval Research Logistics Quarterly, v14, n3, Sept 1967, pp. 391-398.

22. Schrady, David A., Mathematical Models of the Reparable Item Inventory System, Naval Postgraduate School, Monterey, California, NPS 55SO7081A, August 1967, 145 p. (AD 660 371).

23. Sherbrooke, C.C., METRIC: A Multi-Echelon Technique for Recoverable Item Control, The RAND Corporation, RM-5078-PR, November 1966. Also in Operations Research, v16, pp. 122-141, 1968.

24. Sherbrooke, C.C., MINE: Multi-Indenture NORS Evaluation, RM-5826-PR, The RAND Corp., Dec 1968, 35 pp.

25. Simon, Richard M., Stationary Properties of a Two-Echelon Inventory Model for Low Demand Items, The RAND Corp., RM-5928-PR, May 1969, 37 pp.

26. Veinott, Arthur F., "The Status of Mathematical Inventory Theory," Management Science, v12, n11, July 1966, p. 745-777.

27. Von Lanzenaur, Christopher Haehling, Production and Employment Scheduling in Multi-Stage Production Systems, Naval Research Logistics Quarterly, v17, n2, June 1970, pp. 193-198.

28. Wagner, D.E. and E.M. Shetty, "Solution to a Production Scheduling Problem with Fixed Costs," Operations Research Quarterly, v13, n1, 1962.

29. Wagner, H.M., Principles of Operations Research, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1969.

30. Wagner, H.M. and T.M. Whitin, "A Dynamic Version of the Economic Lot Size Model," Management Science, v5, 1958, pp. 89-96.

31. Zangwill, W.I., "A Deterministic Multi-Period Production Scheduling Model with Backlogging, " Management Science, v13, 1966, pp. 105-119.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 77 - 1294	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ALGORITHMS FOR COMPUTING REPAIR SCHEDULES IN PROBABILISTIC ENVIRONMENTS		5. TYPE OF REPORT & PERIOD COVERED Interim 6. PERFORMING ORG. REPORT NUMBER WP76-3011-03
7. AUTHOR(s) W. Steven Demmy		8. CONTRACT OR GRANT NUMBER(s) AFOSR 76-3011 new
9. PERFORMING ORGANIZATION NAME AND ADDRESS Wright State University Department of Administrative Science & Finance Dayton, OH 45431		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, DC 20332		12. REPORT DATE April 1977 13. NUMBER OF PAGES 26
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

repair schedules, algorithm, minimum cost model, support level algorithm

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper, ~~we~~ considers an organization that repairs or overhauls a single type of complex equipment such as aircraft transmitters, navigational computers, or electrical generating units. The organization inventories the repaired assets to meet probabilistic demands for serviceable items. Additional serviceable stocks may also be purchased from outside vendors if the repair process does not satisfy all demands.

The paper considers the problem of computing jointly optimal procurement and repair schedules for such a system. Cost elements (CONTINUED ON REVERSE)

20. Abstract

explicitly considered include set-up costs, learning curve effects, expediting and inventory holding costs, and penalties for incurring shortages or for modifying the schedule from previously established levels. Multi-period lead times, obsolescence and time-discounting effects are also considered.

The paper presents two solution procedures for this scheduling problem. First, we present a general dynamic programming formulation that involves two state variables and two decision variables for each planning period. Next, we present the Support Level Algorithm (SLA) which determines an optimal "Manne-type" schedule. In two important special cases, the SLA simplifies to a form quite similar to the well-known Wagner-Whitin algorithm.